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Conserved dynamics of a two-dimensional random-field model

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We present results from a numerical study of the growth of domains in two dimensions, following a low-temperature quench, in a time-dependent Ginzburg-Landau model in the presence of a quenched random field. The order parameter is *conserved* during the temporal evolution of the system. We find that, at late times, the domains grow logarithmically in time, consistent with studies done for a nonconserved order parameter. We present clear evidence for a breakdown of dynamical scaling of the structure factor. We also demonstrate a crossover behavior exhibited by the tail of the structure factor—the standard Porod-like behavior changes over to a "polymerlike" behavior as the strength of the randomness is increased.

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Binary fluid mixtures in random porous media can be regarded as physical realizations [1,2] of random-field Ising models (RFIM's) whose dynamics is constrained by the conservation of the order parameter. Although this random-field description has been criticized as being inapplicable [3–6] in the case of low-porosity media (such as Vycor glasses), and an alternative "single-pore" description [3] has been presented, this single-pore description, however, cannot be used for phase separation in various gels that resemble a dilute network of strands. It is possible that phase separation in these low-density porous media may exhibit random-field behavior [4].

Quenched random fields pin domain walls and thus impede the growth of ordered domains following a sudden temperature quench. This "pinning" changes the asymptotic growth law from an algebraic to a logarithmic function of time. This has been established by a variety of analytical [7–10], numerical [11–14], and experimental [15] studies in the case when the order parameter is not conserved in the dynamics. In this Rapid Communication we investigate the effect of order-parameter conservation on the asymptotic dynamics of domains pinned by quenched random fields. One of our specific questions is, does the conservation law act as a relevant constraint, further slowing down the logarithmic growth?

General arguments of the kind put forward by Lai, Mazenko, and Valls (LMV) [16] suggest that for quenched random systems, the conservation law does not alter the asymptotic logarithmic growth. LMV suggest that at late times the domain size $R(t)$ grows as

$$\frac{dR}{dt}=\frac{a(R,T)}{R} \tag{1}$$

for the curvature-driven nonconserved case, while for spinodal decomposition

$$\frac{dR}{dt}=\frac{a(R,T)}{R^2}\ , \tag{2}$$

where $a(R,T)$ is related to free-energy barriers required to coarsen a domain of size R at a temperature T. For class-3 systems (in the classification scheme of LMV), which includes the RFIM, $a(R,T)=a_0 e^{-f_0 R/T}$, where f_0 is a free-energy barrier per unit length. Thus the long-time dynamics of domain growth is dominated by "hopping over barriers" arising from interfacial pinning due to the random field. Introducing this form of $a(R,T)$ in the above expressions and integrating, one finds that at late times both the conserved and the nonconserved cases show an asymptotic logarithmic growth $R(t)\sim \ln t$.

In this paper we carry out a numerical analysis of the conserved dynamics of a "soft-spin" version of the RFIM, to study the asymptotic and the preasymptotic growth of the ordered domains. Computational limitations force us to restrict our investigation to two dimensions. This restriction, however, produces an unexpected surprise. We find that the nature of the "shape" of the growing domains changes as the strength of the random field increases. This is reflected in a crossover behavior exhibited by the structure factor for large wave vectors. This surprising crossover is specific to two dimensions. We note that in two dimensions the RFIM does not possess long-range order even at zero temperature. Thus complete phase separation does not proceed following a low-temperature quench—the domains grow up to a finite size R_∞, which increases with decreasing temperature or strength of the random field. This, we shall see, manifests into a breakdown of dynamical scaling behavior of the structure factor at late times.

Consider a conserved order parameter $\psi(\mathbf{x},\tau)$, which evolves according to the time-dependent Ginzburg-Landau (TDGL) model [17] (also called the model-B or the Cahn-Hilliard model in the context of spinodal decomposition),

$$\frac{\partial\psi(\mathbf{x},\tau)}{\partial\tau}=\Gamma\nabla^2\left[\frac{\delta F}{\delta\psi(\mathbf{x},\tau)}\right]+\eta(\mathbf{x},\tau)\ , \tag{3}$$

where Γ is a constant mobility and $\eta(\mathbf{x},\tau)$ is a Gaussian distributed (conserved) random noise term of mean zero and variance proportional to the temperature T. The coarse-grained free-energy functional F of the "soft-spin" version of the RFIM is given by

$$F[\psi(\mathbf{x},\tau)]=\frac{1}{2}\int d^dx\left\{K(\nabla\psi)^2-b\psi^2+\frac{u}{2}\psi^4-2H(\mathbf{x})\psi\right\}, \tag{4}$$

where b, u, and K are phenomenological positive parameters and the quenched random field $H(\mathbf{x})$ has a Gaussian distribution with mean zero and variance $\langle H(\mathbf{x})H(\mathbf{x}')\rangle=H^2\delta(\mathbf{x}-\mathbf{x}')$. We rescale variables so as to express the dynamical equations in a simple form. In terms of a time $t=2\tau\Gamma b^2/K$, a position $\mathbf{r}=(b/K)^{1/2}\mathbf{x}$ and an order parameter $\phi=(u/b)^{1/2}\psi$, the evolution equation simplifies to

$$\frac{\partial\phi(\mathbf{r},t)}{\partial t}=\frac{1}{2}\nabla_r^2[-\nabla_r^2\phi(\mathbf{r},t)-\phi(\mathbf{r},t)+\phi^3(\mathbf{r},t)-h(\mathbf{r})]+\sqrt{\epsilon}\xi(\mathbf{r},t), \tag{5}$$

where $\xi(\mathbf{r},t)$ is the new noise term and the rescaled quenched random field $h(\mathbf{r})$ is given by

$$\langle h(\mathbf{r})h(\mathbf{r}')\rangle=h^2\delta(\mathbf{r}-\mathbf{r}'), \tag{6}$$

with $h^2=uH^2/(b^2K)$. The quantity ϵ, proportional to the temperature $(=uk_BT/Kb)$ and h are the only parameters in the model.

We have integrated the dynamical equation (5) using an Euler scheme with spatial mesh size $\Delta r=1$ and time step $\Delta t=0.025$, which allows us to iterate Eq. (5) up to times $t\le t_{\max}=10\,000$ for a system of size 128^2. We have checked that for smaller time steps the domain size does not change appreciably. For each value of h and ϵ, we perform 20 runs to average over different realizations of the (i) initial configurations of the order parameter, (ii) quenched random field $h(\mathbf{r})$, and (iii) thermal noise $\xi(\mathbf{r},t)$. We compute the circularly averaged pair correlation function $g(r,t)$, and its Fourier transform, the structure factor $S(k,t)$. The size of the domain $R_g(t)$ is computed from the first zero of $g(r,t)$. We find that for times up to $t_{\max}$, $R_g(t)$ is sufficiently smaller than the linear size of the system and no finite-size effects are visible.

In order to study the asymptotic temporal behavior of the domain size, we plot $R_g(t)$ vs $\ln t$ in Fig. 1, for various values of the random field h and for a quench to $\epsilon=0.3$, which corresponds to $\epsilon/\epsilon_c(h=0)\approx0.4$ [18]. For this value of ϵ, the $h=0$ curve reproduces the behavior of the pure system [17,19–21] $R\sim t^a$, where $a=0.30\pm0.02$. Although we find a clear logarithmic behavior for $h=2$, for smaller h values there is a large preasymptotic region where the domain size grows as $R(t)\sim(\ln t)^2$ (Fig. 2). At later times, the growth crosses over to $R\sim\ln t$ behavior. This picture is consistent with the analytical [7–9] and numerical [12–14] studies of the two-dimensional RFIM *in the absence of the conservation law.* Our results thus indicate that the conservation law does not alter domain growth laws in two-dimensional random-field models.

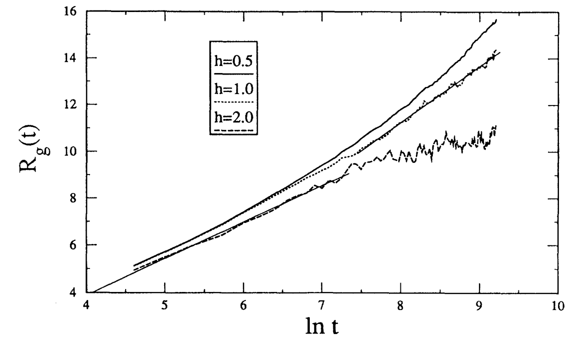

FIG. 1. $R_g(t)$ vs $\ln t$ for several random-field strengths. For $h=2$, the $\ln t$ behavior is clear, whereas for smaller field strengths a preasymptotic $(\ln t)^2$ behavior is found, as shown in Fig. 2. The straight lines are best fits to the data.

This is an important result since the conservation law *changes* the universality class for pure systems.

We point out that the domain size $R(t)$ does not grow indefinitely with time and is expected to saturate to a value [8] $R_\infty(h)\sim\exp[K(\sigma/h)^{4/3}]$, where σ is the surface tension and K is a constant. This is because $d=2$ is the lower critical dimension of the RFIM. In domain-growth problems governed by a stable zero-temperature "discontinuity" fixed point [19], R plays the role of the "divergent correlation length." Only in the vicinity of this fixed point does one observe dynamical scaling. In the present problem, there is no stable zero-temperature "discontinuity" fixed point and no concomitant divergent length scale. Thus the system never enters the true scaling regime. We therefore expect a violation [11,14] of the dynamical scaling of the structure factor $S(k,t)$. Figure 3 explicitly demonstrates this breakdown of dynamical scaling of the circularly averaged structure factor for $h=1$. The deviation from scaling is expected to be most prominent in the region $kR(t)\ll1$, i.e., at length scales larger than the domain size. This is seen quite clearly in Fig. 3.

Further analysis of the asymptotic form of $S(k,t)$ for large k leads to an interesting crossover phenomenon reflecting a change in the morphology of the growing

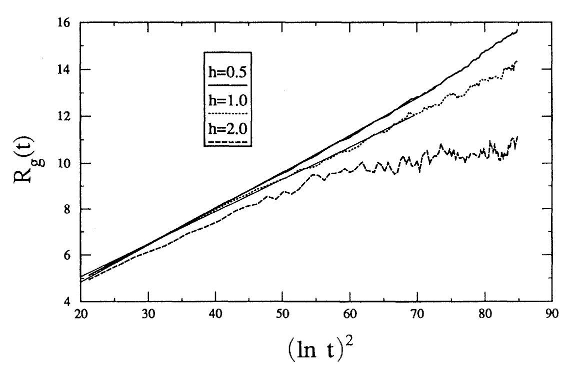

FIG. 2. $R_g(t)$ vs $(\ln t)^2$ for several random-field strengths. This preasymptotic growth law is seen clearly over a long period of time for smaller values of field strengths ($h=0.5$ and $h=1$). The straight lines are best fits to the data.

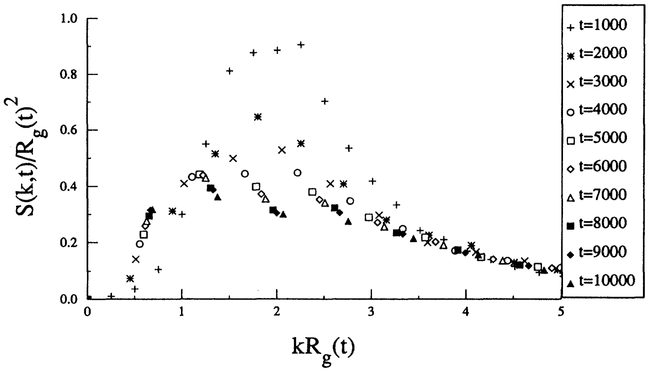

FIG. 3. Demonstration of the breakdown of dynamical scaling behavior for the structure factor $S(k,t)$ for $h=1$. Note that the breakdown is most prominent for small values of $kR_g(t)$.

domains. As shown in Fig. 4, when the randomness is "weak" ($h=0.5$), we find that the tail of $S(k,t)$ goes as the usual k^{-3} in two dimensions. This behavior, known as Porod's law [22], implies that, at late times, the domains grow as compact, two-dimensional regions with sharp interfaces, as shown in Fig. 5(a). On the other hand, for "stronger" randomness ($h=2.0$), we find in Fig. 4 that the asymptotic $S(k,t)\sim k^{-2}$ for large k, suggesting that the domains are no longer compact [see Fig. 5(b)].

How do we understand this crossover phenomenon? At equilibrium, the interface of a domain of size R is roughened by the presence of the random field. The width of the interface ω scales [23] as a power of R, $\omega \approx (R/a)^{\zeta} b$, where a is a microscopic length scale. The diffusivity b, which has dimensions of length, is a monotonically increasing function of the quenched randomness h. The roughening exponent [8,24] $\zeta=(5-d)/3$ in d dimensions, and so, for $d=2$, $\omega \approx (R/a)b(h)$. Since $b(h)$ is an increasing function of h, there is a crossover length scale $b^* \equiv b(h^*) \approx 1$, where $\omega \approx R$. Thus for $h \ll h^*$, the clusters are "compact" and we get a Porod-law behavior [note that for small h, the asymptotic R goes as $\exp(A/h^{\alpha})$, where $\alpha=\frac{4}{3}$]. For $h \gg h^*$ and $\zeta=1$, the interface is so convoluted as to "fill out" a bulk two-dimensional "volume." Considering a circle of radius R, this implies that the "density" of interface within the circle is uniform. Since the pair correlation function $g(r,t)$ is a measure of this density for $r \ll R$, its Fourier transform $S(k,t)$ should go as k^{-2} for $kR \gg 1$. It should be noted that this is exactly the form obtained for an ideal (Gaussian) polymer chain [24] in arbitrary dimensions.

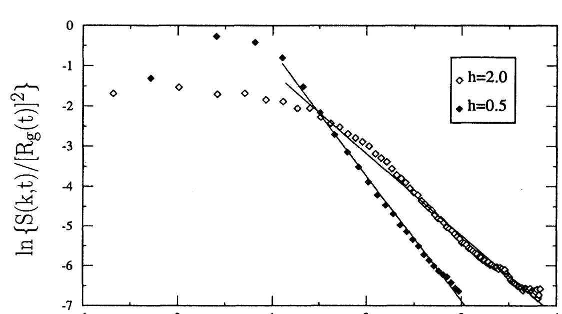

FIG. 4. Violation of Porod's law for large disorder. At late times ($t=10\,000$) the tail of $S(k,t)\sim k^{-3}$ for $h=0.5$, consistent with Porod's law, whereas $S(k,t)\sim k^{-2}$, resembling a Gaussian polymer chain, for $h=2$.

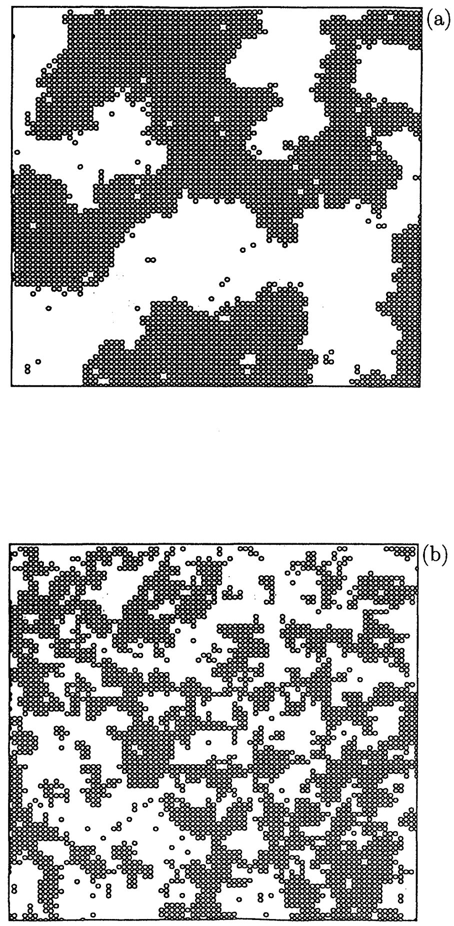

FIG. 5. (a) A typical configuration of the system (80×80 section) at late time ($t=10\,000$) for $h=0.5$. Note the sharp interfaces. (b) Same as (a), except that here $h=2.0$. Note the convoluted interfaces.

To summarize, we have numerically integrated the evolution equations for a "soft-spin" version of the RFIM in two dimensions where the order parameter is conserved. Analysis of the equal-time correlation function $g(r,t)$ shows a preasymptotic $(\ln t)^2$ growth law followed by an asymptotic $\ln t$ growth of the domain size. We provide clear evidence of a breakdown of dynamical scaling in the region $kR(t) \ll 1$. A more detailed analysis shows a crossover from the growth of compact domains to polymerlike domains as the degree of randomness increases, as evidenced by the shape of the structure factor for large wave vectors. This indicates the presence of a dynamical analogue of a "disordered line" in the phase diagram of the two-dimensional RFIM.

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